Euclid's Proof

Recall that we know that every integer can be written as a product of prime numbers. In particular, every integer greater than 1 has a prime divisor.

Proposition: There are infinitely many primes.

Assume there only finitely many.

$p_1 p_2 \cdots p_N$ for some $N \in \mathbb{Z}$

$\exists N \in \mathbb{Z}$ ~~s.t. if p is prime~~

and primes $p_1, \ldots, p_N$ so that

if q is prime then $q \in \{p_1, \ldots, p_N\}$.

$E = p_1 p_2 \cdots p_N + 1$

E is not divisible by any of $p_1, \ldots, p_N$.

E is a product of primes.

E is prime or it has

Euclid's Proof cont'd

Proof: Suppose that there are only finitely many prime numbers.

- Multiply them together and let P be their product.
- Consider the integer $P + 1$.
- This integer must have a prime divisor p, which must be greater than one, so we can write $P + 1 = pA$.
- Since P is the product of all the primes, we know that p is a divisor of P, so we can write $P = pB$.
- Therefore $1 = pA - P = pA - pB = p(A - B)$.
- This implies that p is a divisor of 1, so $p = 1$.
- We've proved that $p > 1$ and $p = 1$, which is a contradiction. Thus there cannot be finitely many primes.

$2 \cdot 3 + 1 = 7$

$2 \cdot 5 \cdot 3 + 1 = 31$

$2 \cdot 3 \cdot 5 \cdot 7 + 1 = 211$

Harmonic Series

$\sum \frac{1}{n}$ diverges

$\sum_{p \text{ prime}} \frac{1}{p}$ diverges

$\log \log n = 6$

$n \sim e^{e^6}$

$\frac{\pi(x)}{x} = \frac{\#\text{primes} \leq x}{x} \sim \frac{1}{\log x}$